

Chapter 8: Power, sound intensity, and the decibel scale

In this chapter, I will discuss how we quantify sound intensity. It will take a few steps to get there. Mathematically, this can be the trickiest stuff in the course, especially if you haven't dealt with logarithms in a while. A review of the relevant math can be found in the appendix available with this coursepack. I'll cover what power means, and relate that to intensity. We have to know how sound intensity drops off as you move further away from a source. Then, I introduce the decibel scale, which is a logarithmic scale designed to make a wide range of sound intensities easier to talk about.

Power and intensity

We quantify loudness by speaking of a sound's *intensity*. There isn't a perfect correlation between loudness and intensity. Imagine playing a 30,000 Hz sound through a high-frequency speaker turned up as high as it will go. You certainly won't perceive loudness in the usual sense because that frequency lies outside the range of human hearing. For sounds within our range of hearing (again, typically 20 Hz – 20 kHz), there is some correlation between intensity and loudness. I will now specify what I mean by intensity, but we have to review some old concepts first.

In our mechanics chapter, we discussed energy. A few of the energy types we considered were *potential energy* (the energy associated with an object's position) and *kinetic energy* (the energy associated with motion). Energy can be transferred or used up at different rates, however, and that rate of change or usage is called *power*.

Discussion/reading question: What objects do you know of that are often described by their power level?

We saw earlier that gravitational potential energy depends on height, so we know as an elevator cable lifts an elevator, it gives the elevator potential energy. However, the cable (according to what the motor is doing) can lift quickly or slowly. That is a power difference; lifting something quickly takes more power than doing it slowly.

Perhaps by now you've recalled that things like light bulbs, car engines, or perhaps speakers/amplifiers are commonly given power ratings. Car engines have a horsepower rating, and light bulbs and other electronic devices are rated in Watts (W). The definition of a Watt is as follows:

$$1 \text{ Watt} = 1 \frac{J}{s}$$

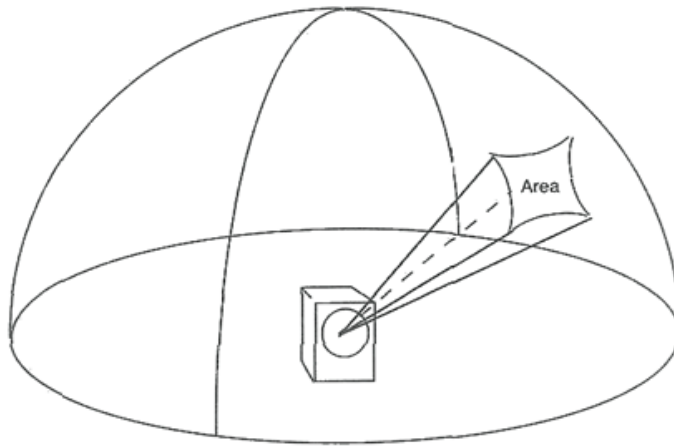
Also, 1 horsepower (hp) = 746 W. Incandescent light bulbs range from 40-100 W typically, and their compact fluorescent counterparts are often less than half the wattage of their incandescent equivalents.

Intensity, then, describes how the energy put out by a source spreads out. You should intuitively know that a speaker, for example, sounds louder the closer you get to it. That's because if you are closer to a source of energy, it has less room to spread out. A speaker putting out power to an area might look like the figure below.

Mathematically, intensity is defined as follows:

$$Intensity = \frac{power\ output}{surface\ area}$$

We assume the surface area is that of an imaginary sphere and that the energy is dispersed uniformly in all directions.



A source putting out sound over an area ([source](#))

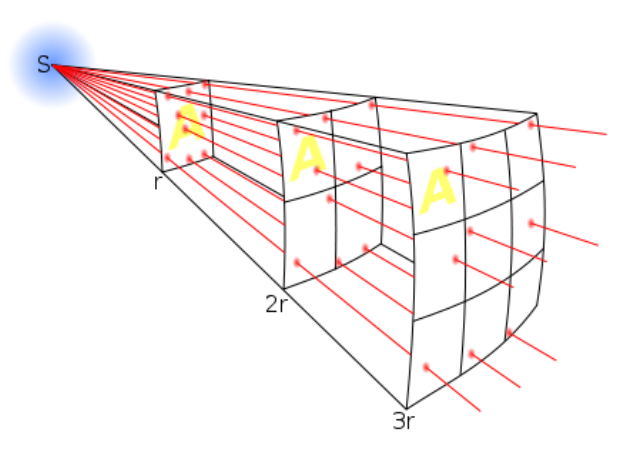
Using the fact from geometry that the surface area of a sphere is $4\pi r^2$, we can rewrite intensity like this:

$$I = \frac{P}{4\pi r^2}$$

This equation gives intensity as a function of the power the source puts out and the distance r the listener is from it. Without the presence of a listener at a specific location, intensity calculations are meaningless since listeners at different distances perceive different intensities.

The key feature of this equation is the r^2 term in the denominator. This term describes how intensity drops off with distance. Specifically, this mathematical relationship is called the *inverse square law*: the intensity put out by an energy source is inversely proportional to the square of the distance from the source.

Proportional reasoning can be tricky, but perhaps not if you visualize it. Consider the following image of a source putting out energy:



A picture of how energy spreads from a source ([source](#))

Picture the red lines as energy the source is putting out in a particular direction. At a distance r , all that energy goes into one square worth of area. At a distance $2r$, that same energy is spread over four times the area, and at a distance $3r$, that same energy is spread over nine times the area. Therefore, since intensity represents power output divided by a surface area, the intensity at $2r$ is four times less than it is at r , and the intensity at $3r$ is nine times less than it is at r .

The units of intensity in the SI system are watts per square meter (W/m^2). That should make sense since watts are the unit of power and square meters are the unit of area.

Examples of the inverse square law in action

Example: If a source puts out power such that the intensity 1 m away from it is 24 W/m^2 , what is the intensity 2 m away? 3 m?

Answer: Moving double the distance away drops the intensity by a factor of 2^2 , or 4. Therefore, at 2 m, the intensity is $24/4 \text{ W/m}^2$ or 6 W/m^2 . Likewise, at 3 m, the intensity is $24/9$ (about 2.67) W/m^2 .

Example 2: If a speaker's intensity at a distance of 8 m away is 0.01 W/m^2 , what is the intensity 1 m away? 3 m away?

Answer: You can work backward this time. The intensity at a distance of 8 m is 64 times LESS than the intensity at 1 m (think inverse square law again!) Therefore, the intensity put out by the speaker at 1 m distance is $0.01 \times 64 = 0.64 \text{ W/m}^2$. Then, to get the intensity at 3 m distance, *divide* this figure by 9. 0.64 divided by 9 is about 0.071 W/m^2 .

OK, we have a good measure of intensity – why use logs?

The human ear is a versatile and sensitive organ! A normal conversation might have one million times more intensity than the softest sounds we can hear. A loud rock concert may be one million times more intense than a conversation. That's a factor of one trillion between the quietest sound we can hear and one of the loudest. Rather than dealing with all these large powers of ten, we use logarithms to simplify the scale of intensity somewhat. That's what the decibel scale does.

The ear is most sensitive in the region around the [region between 2-5 kHz](#). In those frequencies, the lowest intensity sound most people can perceive is around 10^{-12} W/m^2 . This intensity is called the *threshold of human hearing*, and it's given the symbol I_0 . The bel and decibel scale are based on taking the ratio of any given sound intensity with the threshold of human hearing, and using the log of that.

Bels and decibels

The bel (B) gets its name from Alexander Graham Bell, the inventor of the telephone and founder of Bell Labs. It is defined as the log of the ratio between a given signal or intensity and some reference amount. In acoustics and hearing, the reference level is the intensity corresponding to the threshold of human hearing. Therefore, the number of bels corresponding to an intensity I is given by:

$$B = \log \left(\frac{I}{I_0} \right)$$

What that means is if a given sound is one million times more intense than the threshold of hearing, the number of bels is 6. If a sound is a thousand times more intense than the threshold of hearing, the number of bels is 3. Historically, bels are rarely used, and the smaller unit called the *decibel* is preferred. There are ten decibels in a bel, so the number of dB corresponding to an intensity I is given by:

$$\beta = 10 \log \left(\frac{I}{I_0} \right)$$

The Greek letter beta (β) is a standard variable used in the literature for decibels. Again, dB measurements are worthless without some reference level, and for sound intensity, we set $I_0 = 10^{-12} \text{ W/m}^2$, the threshold of human hearing.

Units and things to look out for: When using the decibel scale, always write dB as your unit if you've gone through this logarithm formula. Despite the fact that dB is a unit, decibels are a *dimensionless* quantity. Also, you cannot simply double a sound intensity by doubling the number of decibels. This is a mistake common to those who have not dealt with logarithmic scales before. You CAN double the intensity if you express it in units of W/m^2 , but then you have to convert THAT result to decibels.

Example: A loud radio puts out an intensity of 10^{-5} W/m^2 2 m away from it. (a) How many dB is that? (b) What's the intensity in W/m^2 and dB if you are the same distance from TWO of these radios?

To get the intensity of one radio, use the decibel formula:

$$\beta = 10 \log \left(\frac{10^{-5} \frac{W}{m^2}}{10^{-12} \frac{W}{m^2}} \right) = 10 \log(10^7) = 10 (7) = 70 \text{ dB}$$

(b) Doubling the intensity in W/m^2 just means you multiply the given intensity by two, so two radios at the same distance away would make an intensity of $2 \times 10^{-5} W/m^2$. Now, we put *this* intensity in the dB formula:

$$\beta = 10 \log \left(\frac{2 \times 10^{-5} \frac{W}{m^2}}{10^{-12} \frac{W}{m^2}} \right) = 10 \log(2 \times 10^7) = 10 (7.3) = 73 \text{ dB}$$

Therefore, doubling the intensity of a source doesn't double the number of decibels, it increases the number of decibels by three. This is true no matter *what* intensity you start with, and it has to do with the rule for taking the log of a product:

$$10 \log(2x) = 10 (\log 2 + \log x) = 10 \log 2 + 10 \log x$$

The log of 2 is about 0.3, so therefore, introducing a factor of two into intensity will add $10 * 0.3$ or 3 dB to your total. Likewise, increasing intensity by a factor of four adds 6 dB (two doublings), increasing intensity by a factor of eight adds 9 dB (three doublings), and increasing intensity by a factor of 10 adds 10 dB. In general, increasing intensity in W/m^2 by a factor n adds $(10 \log n)$ to the dB level.

Going backwards: Since the math of the inverse square law and ideas like the doubling trick above depend on the original W/m^2 units for intensity, it is useful to be able to convert dB values to W/m^2 values. We can try that with a 50 dB sound. Filling in some numbers in the dB formula tells us

$$50 = 10 \log \left(\frac{I}{10^{-12} W/m^2} \right)$$

and now we see the issue: the goal is to solve for the intensity in the parenthesis.

$$5 = \log \left(\frac{I}{10^{-12} W/m^2} \right)$$

The way to undo the log is to raise 10 to the power of each side—see the math supplement Appendix for details on that. Here's what that looks like:

$$10^5 = \left(\frac{I}{10^{-12} W/m^2} \right)$$

and then we get $I = 10^5 \times 10^{-12} W/m^2$, or $10^{-7} W/m^2$.

Notice this is 100 times smaller in intensity than the 70 dB sound we dealt with in an earlier example. Two factors of 10 lessening of intensity results in two “-10” changes to the dB level.

Other additions of decibels: If one is careful, intensities can be added. If intensity is written in W/m^2 units, two sources' intensities are added directly. A common temptation is to do the same with decibel levels, and that is not correct.

Example: Two sources near you would independently produce intensity levels of 71 and 48 dB. What is the combined intensity?

These intensities correspond roughly to a loud conversation and a refrigerator hum respectively. If you add the levels, you get 119 dB, which is loud rock concert territory, so that's clearly wrong. But what's correct? First, convert both to SI units. Here's what the 71 dB example looks like:

$$71 = 10 \log \left(\frac{I}{10^{-12} \text{ W/m}^2} \right)$$

$$7.1 = \log \left(\frac{I}{10^{-12} \text{ W/m}^2} \right)$$

$$10^{7.1} = \left(\frac{I}{10^{-12} \text{ W/m}^2} \right)$$

and then the SI intensity is $10^{7.1-12}$ or $10^{-4.9} \text{ W/m}^2$. Calculators in scientific notation mode often have trouble with decimal powers of 10 (in the E notation), but an entry of "10^-4.9" should work just fine. Written in regular scientific notation, that's about $1.26 \times 10^{-5} \text{ W/m}^2$.

Following the same steps with the 48 dB sound, we get an intensity of $10^{-7.2}$ or $6.3 \times 10^{-8} \text{ W/m}^2$.

NOW the intensities can be added directly. When I do that, I get $1.27 \times 10^{-5} \text{ W/m}^2$, and that's after rounding that second decimal place up. It's as though adding the 48 dB sound barely makes a difference, and that's completely true; 48 dB is over 20 dB lower than 71 dB, so it is over 100 times less intense. That's a drop in the bucket, so to speak. If I don't round numbers until the end and then put the final intensity back in the dB formula, I get 71.02 dB, roughly. If it takes a full doubling of intensity to gain 3 dB, it shouldn't surprise you that adding an insignificantly low intensity should barely touch the dB level at all.

Sound pressure levels: In *Science of Acoustics*, there are invariably a few audio students who ask whether the decibel formula should have a 20 in front of it instead of a 10. Another consequence of this is that they'll think a doubling of *something* corresponds to a 6 dB increase, not a 3 dB increase. In one sense, they are right, but not about the dB *intensity* formula. The decibel version of *sound pressure level* is written like this:

$$\beta_{SPL} = 20 \log \left(\frac{p}{p_0} \right)$$

Here, the *ps* represent *pressures*, not intensities, so the quantities used and the reference level are different here. Both equations are correct; intensity varies with the square of pressure, so the relative levels should be the same no matter how you write them. For the curious, a pressure of about 20

micropascals is the relevant threshold here. A pascal (Pa) is the SI unit for pressure and is equivalent to a Newton of force spread over a square meter of area (1 N/m^2). For something the size of our eardrum, the force needed to produce that pressure is on the order of a nanonewton.

Relative decibel levels

The decibel formula discussed above is based on sound intensity level relative to the threshold of human hearing. It is sometimes useful instead to think about the difference in decibel level between two sounds of intensities I_1 and I_2 . I'll use the notation $\Delta\beta$ to represent difference in decibels.

$$\Delta\beta = \beta_2 - \beta_1 = 10 \log\left(\frac{I_2}{I_0}\right) - 10 \log\left(\frac{I_1}{I_0}\right)$$

$$\Delta\beta = 10 (\log I_2 - \log I_0) - 10 (\log I_1 - \log I_0)$$

And since the $\log I_0$ terms cancel,

$$\Delta\beta = 10 (\log I_2 - \log I_1)$$

$$\Delta\beta = 10 \log\left(\frac{I_2}{I_1}\right)$$

Any of the dB shortcuts mentioned can be proven in light of this formula. For example, if one sound I_2 is twice as intense as I_1 , then $I_2/I_1 = 2$, and the difference in dB is $10 \log 2$ or about 3.

The faders pictured below use a kind of relative dB scaling; the “unity gain” level is called 0 dB for reference, and the numbers shown are dB levels relative to unity gain. The homework problems give examples showing how even something like a 12 dB drop with the faders leads to a large percentage of intensity being mixed out.



Some faders. The scale on the right is in dB relative to a “unity gain” level near the top (0 dB)

Recall from above that we may encounter dB formulae with a 20 in front of the logarithm. These are common when finding sound *pressure* levels. The intensity of a sound is proportional to its pressure level squared, so we get, for example:

$$\Delta\beta = 10 \log\left(\frac{I_2}{I_1}\right) = 10 \log\left(\frac{p_2}{p_1}\right)^2 = 20 \log\left(\frac{p_2}{p_1}\right)$$

Sound pressure level is proportional to the voltage recorded digitally by an analog-to-digital converter, which leads us to a way of expressing dynamic range in decibels.

Dynamic range

People who study acoustics (especially in this class) are often interested in digital recording applications. A digital audio workstation uses analog-to-digital conversion (ADC) to convert analog acoustic signals to digital voltage levels that can be recorded by a computer. Computers are digital machines that use binary arithmetic for processing, so we need to discuss this for a minute:

Normally, we count in base-10 (decimal) arithmetic, meaning numbers are written in terms of a ones place, tens place, hundreds place, and so on. For example, the decimal number 24601 is written:

Decimal place	Ten thousands	Thousands	Hundreds	Tens	Ones
	2	4	6	0	1

And written out the long way, this means $2 \times 10,000 + 4 \times 1,000 + 6 \times 100 + 0 \times 10 + 1 \times 1$. The digits 0 through 9 are the possibilities that can go in each decimal place.

Note that, say, for two-digit numbers, the largest number we can write is 99, which equals $10^2 - 1$. Likewise, the largest three-digit number is 999, which equals $10^3 - 1$.

Binary digits are written with a different set of binary places. Instead of ones, tens, hundreds, etc., we use ones, twos, fours, eights, etc. The only digits possible are 0 and 1 – these correspond to “on” and “off” positions if you want to think of this electronically. Suppose we have a binary number 10110. To convert this to decimal, consider this breakdown:

Decimal place	Sixteens	Eights	Fours	Twos	Ones
	1	0	1	1	0

Then we can convert – this binary number equals $1 \times 16 + 0 \times 8 + 1 \times 4 + 1 \times 2 + 0 \times 1 = 16 + 4 + 2 = 22$.

To go the other way, keep thinking of the largest power of two less than the number you want. For example, to convert the decimal number 40 to binary, think of the powers of two; the largest one less than 40 is 32. That would leave 8 remaining, and eight is also a power of two.

$40 = 32 + (\text{no sixteens}) + 8 + (\text{no fours}) + (\text{no twos}) + (\text{no ones}) \rightarrow 101000$ is the binary equivalent.

Consider the largest 4-bit binary number: 1111. In decimal, this is $8 + 4 + 2 + 1 = 15$. Notice that $15 = 2^4 - 1$. In general, the largest N-digit binary number is $2^N - 1$.

What an ADC does is take some fixed voltage range and break it into pieces. The number of pieces we can divide this range into depends on the number of bits. For 16-bit ADC, we can use $2^{16} - 1$ pieces. Therefore, the approximate ratio between the largest (V_2) and smallest (V_1) signal we can process is about 2^{16} . The *dynamic range* for 16-bit ADC is thus:

$$\Delta\beta = 20 \log \left(\frac{p_2}{p_1} \right) = 20 \log \left(\frac{V_2}{V_1} \right) = 20 \log 2^{16} = 96 \text{ dB}$$

A similar calculation will work for different numbers of ADC bits (just replace the 16).

Examples of decibel intensity levels

Some of these are taken from [this website](#):

Source	dB	Source	dB
Rustling leaf	10	Power saw at 3'	110
Whispering	30	Rock concert	115-120
Normal conversation	60	Pain threshold	125-130
Train whistle at 500'	90	Jet engine at 100'	140
Subway train at 200'	95	Death of hearing tissue	180

Between a leaf rustle and a jet engine, there are thirteen powers of ten difference in intensity (ten trillion). Of course, you should wear ear protection as you get to the higher-intensity examples I listed, but it just goes to show the wide range of sound intensities present in our environment.

Summary

- The human ear is capable of processing a wide range of sound intensities.
- There is a correlation between intensity and loudness provided the sound is within our frequency range.
- Intensity is defined as power spread over an imaginary surface area, and it has the SI unit W/m^2 .
- Intensity from a source falls off according to the *inverse square law*.
- Since there are many powers of ten difference between the loudest and quietest sounds, a logarithmic scale called the decibel (dB) scale is used to make the numbers easier to discuss without large decimals.
- Doubling an intensity ADDS 3 to its dB level, and multiplying an intensity by ten ADDS 10 dB to its level.

Homework problems

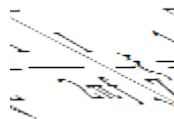
- Complete the following intensity tables. The distances represent the distance you are from the source.

Distance (m)	1	2	3	4	5	10
Intensity (W/m ²)	24					
Distance (m)	1	2	3	4	5	10
Intensity (W/m ²)				0.2		

- 100 typewriters make a combined sound of intensity 10^{-5} W/m^2 . (a) Convert this 10^{-5} W/m^2 sound to dB, (b) Give in W/m^2 **and** dB the intensity produced by **one** such typewriter.
- In *The Music Man*, one of the main songs talks about how 76 trombones led the big parade. If one trombone has an intensity of 80 dB, (a) find its intensity in W/m^2 , (b) find the intensity of 76 trombones, and (c) find the dB level of all 76. How many more dB do 76 trombones put out than just one?
- Find the log of 76 and multiply it by 10. If you did #3 right, you should find this number in that problem. Why?
- The decibel rules of thumb can be combined. (a) If a sound has intensity x dB, how many dB does a sound 100 times more intense have? (b) If another sound has intensity y dB, how many dB does a sound 4 times less intense have? (c) Combine what you know about (a) and (b): If a sound has intensity z dB, how many dB does a sound 25 times more intense have? (Hint: stack your answers to (a) and (b).)
- A decibel meter reads a sound intensity of 80 dB when you stand 2 m from a loudspeaker. What will the meter read (in dB) if you stand 40 m away?
- You are standing at a distance R from a small source of sound (of size much less than R , so you don't have to worry about, say, the size of the speaker) at ground level, out in

the open where reflections may be neglected. The sound level is L in dB. If you now move to a distance nR (where n is any positive number), what is the new sound level in dB?

8. Three sounds at a standard 1000 Hz frequency have intensity levels of 65 dB, 68 dB, and 76 dB. What will be their combined sound intensity level in dB?
9. Prove that if you play two sounds of identical frequency f and intensity I the resulting sound will be 3 dB louder than for a single sound.
10. (a) If you attend a rock concert wearing earplugs that provide a reduction of 13 dB, what is the ratio of the sound intensity reaching your eardrums to the intensity just outside the plugs? (b) What percentage of the sound energy is getting blocked out? (c) What percentage of the incoming sound intensity would have to be blocked out to get a 23 dB reduction?
11. An outdoor air-raid siren produces a sound level of 115 dB at a distance of 10 m. How far away from the siren must you go to find a level of 85 dB? (assume no reflecting surfaces or other complications)
12. Suppose you want to know how far a speaker has to move to create a loud 100 dB sound wave of frequency 1000 Hz. To do so, we can use an expression found in the work of musician, university lecturer, and sound engineer Eberhard Sengpiel (who died in 2014). The physical displacement of air needed (and called ξ) is given by



where I is the intensity in W/m^2 and Z is the acoustic impedance. For unimpeded air at 20°C , take $Z = 413 \text{ N s / m}^3$. (a) Calculate ξ under these conditions. (b) What would be the displacement for a 100 Hz sound at the same dB level? (c) Now consider a 40 dB sound at 1000 Hz. What's the displacement in this case? Compare to the width of a couple of atoms ($\sim 1 \text{ nm}$).